Kinematics Equations

1. (a) $v_i = 15.0 m/s$
$v_f = 19.5 m/s$
$t = 4.0 s$
$a = ?$

$v_f = v_i + at$
$19.5 = 15 + a(4)$
$\underline{a = 1.1 m/s^2}$

(b) $d = ?$

$d = \left(\frac{v_i + v_f}{2}\right)t$
$= \left(\frac{15 + 19.5}{2}\right)4$
$= \underline{69 m}$

(c) $V_i = 4.6 m/s$
$a = -0.64 m/s^2$
$v_f = -2.3 m/s$
$d = ?$

$v_f^2 = v_i^2 + 2ad$
$(-2.3)^2 = (4.6)^2 + 2(-0.64)d$
$-15.87 = -1.28 d$
$\underline{d = 12.4 m}$

(d) $V_i = 14.0 m/s$
$a = 2.0 m/s^2$
$t = 3.0 s$
$d = ?$

$d = v_i t + \frac{1}{2}at^2$
$= 14(3) + \frac{1}{2}(2)(3)^2$
$= \underline{51 m}$

(e) west is −ve.
$v_i = -18 km/h = -5 m/s$
$v_f = 21.6 km/h = 6 m/s$
$a = .2 m/s^2$
$d = ?$

$v_f^2 = v_i^2 + 2ad$
$(6)^2 = (-5)^2 + 2(.2)d$
$\underline{d = 27.5 m}$

(f) $v_i = 80 km/h = 22.2 m/s$
$v_f = 160 km/h = 44.4 m/s$
$a = 2.0 m/s^2$
$t = ?$

$v_f = v_i + at$
$44.4 = 22.2 + 2t$
$\underline{t = 11.1 s}$

1.(g) $v_i = 5.6\ m/s$
$d = 24.0\ m$
$t = 2.0\ s$
$v_f = ?$

$$d = \left(\frac{v_i + v_f}{2}\right)t$$

$$24 = \left(\frac{5.6 + v_f}{2}\right)(2)$$

$v_f = 18.4\ m/s$ or $66.2\ km/h$

(h) $v_i = 2.2\ m/s$
$a = 3.6\ m/s^2$
$d = 12.0\ m$
$v_f = ?$

$$v_f^2 = v_i^2 + 2ad$$
$$= (2.2)^2 + 2(3.6)(12)$$
$$\sqrt{v_f^2} = \sqrt{91.24}$$
$v_f = 9.55\ m/s$

(i) $v_i = 34\ m/s$
$a = -2.0\ m/s^2$
$t = 7.2\ s$
$v_f = 0$
$d = ?$

$$d = \left(\frac{v_i + v_f}{2}\right)t$$
$$= \left(\frac{34}{2}\right)7.2$$
$d = 122.4\ m$

(2)

30 s	8 s	12 s
$a = 0$	$v_i = 12\ m/s$	$a = 0$
$v = 12\ m/s$	$a = 1.5\ m/s^2$	$v = 24\ m/s$
$t = 30.0\ s$	$t = 8.0\ s$	$t = 12\ s$
	$d = ?$	

$$v = \frac{d}{t}$$
$$12 = \frac{d}{30}$$
$d = 360\ m$

$$d = v_i t + \frac{1}{2}at^2$$
$$= 12(8) + \frac{1}{2}(1.5)(8)^2$$
$d = 816\ m$

$v_f = ?$
$$v_f = v_i + at$$
$$= 12 + (1.5)(8)$$
$$= 24\ m/s$$

$$v = \frac{d}{t}$$
$$24 = \frac{d}{12}$$
$d = 288\ m$

Total distance = 360 + 816 + 288 = 1466 m

(3) $a = 3.0\ m/s^2$ $a = -5.0\ m/s^2$

$v_i = 20\ m/s$
$a = 3.0\ m/s^2$
$t = 5.0\ s$
$d = ?$

$d = v_i t + \frac{1}{2} a t^2$
$= (20)(5) + \frac{1}{2}(3)(5)^2$
$= 137.5\ m$

$v_f = v_i + at$
$= 20 + 3(5)$
$= 35\ m/s$

$v_i = 35\ m/s$
$v_f = 0$
$a = -5.0\ m/s^2$
$d = ?$

$v_f^2 = v_i^2 + 2ad$
$0 = (35)^2 + 2(-5)d$
$-1225 = -10d$
$d = 122.5\ m$

Total distance = $137.5 + 122.5$ = $\underline{260\ m}$

(4) $v_i = 0$
$a = 7.5\ m/s^2$
$t = 1.2\ s$
$d = ?$
$v_f = ?$

$d = v_i t + \frac{1}{2} a t^2$
$= \frac{1}{2}(7.5)(1.2)^2$
$= 5.4\ m$ ← the sprinter traveled 5.4 m in the first 1.2 s.

$v_f = v_i + at$
$= (7.5)(1.2)$
$= 9\ m/s$ ← the sprinter is going 9 m/s after the first 1.2 s.

4 the rest of the race $250\,m - 5.4\,m = 244.6\,m$.

$a = 0$
$v = 9\ m/s$
$d = 244.6\ m$
$t = ?$

$$v = \frac{d}{t}$$

$$9 = \frac{244.6}{t}$$

$t = 27.2\ s$

Total time for race $1.2 + 27.2 = \underline{28.4\,s}$

(5) (a) $v_i = 33.5\ m/s$
$v_f = 10\ m/s$
$t = 8.05\ s$
$d = ?$

$$d = \left(\frac{v_i + v_f}{2}\right) t$$
$$= \left(\frac{33.5 + 10}{2}\right) 8.05$$
$$= 175.09\ m$$

$v_i = 10\ m/s$
$a = 0$
$t = 12.4\ s$
$d = ?$

$$v = \frac{d}{t}$$
$$10 = \frac{d}{12.4}$$
$$d = 124.0\ m$$

$v_i = 10\ m/s$
$a = 2.35\ m/s^2$
$v_f = 23.8\ m/s$
$d = ?$

$$v_f^2 = v_i^2 + 2ad$$
$$(23.8)^2 = (10)^2 + 2(2.35)d$$
$$566.44 = 100 + 4.7d$$
$$d = 99.24\ m$$

Total displacement $175.09 + 124.0 + 99.24 = \underline{398.3\,m}$

5 (b) we need the time for the third part

$v_i = 10$ m/s
$a = 2.35$ m/s^2
$v_f = 23.8$ m/s
$t = ?$

$v_f = v_i + at$
$23.8 = 10 + 2.35t$
$13.8 = 2.35t$
$t = 5.87$ s.

total displacement = 398.3
total time = 8.05 s + 12.4 s + 5.87 s = 26.32 s

$v = \frac{d}{t}$
$= \frac{398.3 \text{ m}}{26.32}$
$= 15.1$ m/s

(6) (a) $v_i = 0$
$a = 12.3$ m/s^2
$t = 10.2$ s
$d = ?$

$d = v_i t + \frac{1}{2}at^2$
$= \frac{1}{2}(12.3)(10.2)^2$
$= 639.8$ m

once it runs out of fuel, the rocket keeps going up; its initial velocity is the velocity it had when it ran out of fuel.

$v_f = ?$
$v_i = 0$
$a = 12.3$ m/s^2
$t = 10.2$ s

$v_f = v_i + at$
$= (12.3)(10.2)$
$= 125.46$ m/s

as the rocket keeps going up, it slows down because of gravity.

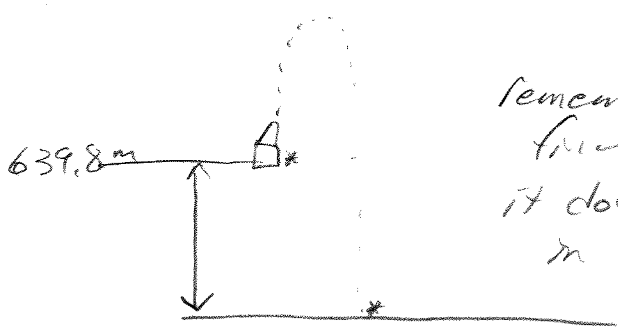
$v_i = 125.46$ m/s
$v_f = 0$
$a = -9.8$ m/s²
$d = ?$

$$v_f^2 = v_i^2 + 2ad$$
$$0 = (125.46)^2 + 2(-9.8)d$$
$$-15740.2 = -19.6d$$
$$d = 803.07 \text{ m}$$

total height = 639.8 + 803.07 = <u>1442.9 m</u>

(b) time while rocket has fuel = 10.2 s
velocity once it runs out of fuel = 125.46 m/s

when the rocket runs out of fuel it is 639.8m high; that means that the displacement of the rocket will be −639.8 m



remember, displacement is final position − intial position it doesn't matter what happens in between.

$v_i = 125.46$ m/s
$d = -639.8$ m
$a = -9.8$ m/s²
$t = ?$

$$d = v_i t + \frac{1}{2}at^2$$
$$-639.8 = 125.46t + \frac{1}{2}(-9.8)t^2$$
$$-639.8 = 125.46t - 4.9t^2$$

(this is a quadratic equation, we need to solve it using the quadratic formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

rearrange equation

$$4.9t^2 - 125.46t - 639.8 = 0$$

$$t = \frac{(125.46) \pm \sqrt{(125.46)^2 - 4(4.9)(-639.8)}}{2(4.9)}$$

$$t = \frac{125.46 \pm 168.167}{9.8}$$

this has two solutions

$$t = \frac{125.46 + 168.167}{9.8} = 29.96\ s \quad \text{or} \quad t = \frac{125.46 - 168.167}{9.8} = -4.36\ s$$

t = 29.96 s must be correct because we can't have negative time.

∴ total time = 10.2 s + 29.96 s = 40.2 s

(7) $v_i = 0$
$d = 645\ m$
$a = 16.0\ m/s^2$
$v_f = ?$

$$v_f^2 = v_i^2 + 2ad$$
$$= 2(16)(645)$$
$$\sqrt{v_f^2} = \sqrt{20640}$$
$$v_f = 143.67\ m/s$$

$v_i = 143.67\ m/s$
$a = -18.2$
$v_f = 0$
$d = ?$

$$v_f^2 = v_i^2 + 2ad$$
$$0 = (143.67)^2 + 2(-18.2)d$$
$$d = 567\ m$$

Since the track is only 500 m long, the sled falls off the track.

(8) $v_i = 0$
$a = 3.8\ m/s^2$
$t = 4.5\ s$
$d = ?$
$v_f = ?$

$$d = v_i t + \frac{1}{2}at^2$$
$$= \frac{1}{2}(3.8)(4.5)^2$$
$$= 38.5\ m$$

$$v_f = v_i + at$$
$$= 0 + 3.8(4.5)$$
$$= 17.1\ m/s$$

for the rest of the race ... $150 - 38.5 = 111.5\ m$
$a = 0$
$v = 17.1\ m/s$
$d = 111.5\ m$
$t = ?$

$$v = \frac{d}{t}$$
$$17.1 = \frac{111.5}{t}$$
$$t = 6.52\ s$$

Total time = $4.5\ s + 6.52\ s = \underline{11.02\ s}$

(10) $v_i = 12\ m/s$
$a = 6.0\ m/s^2$
$d = 63\ m$
$t = ?$

$$d = v_i t + \frac{1}{2}at^2$$
$$63 = 12t + \frac{1}{2}(6)t^2$$
$$3t^2 + 12t - 63 = 0$$

$$t = \frac{-12 \pm \sqrt{12^2 - 4(3)(-63)}}{2(3)}$$
$$= \frac{-12 \pm 30}{6}$$
$$\underline{t = 3\ s}$$

(b) displacement was x, so solve for x

$$x = 1.8t^2$$
$$x = 1.8(15.4)^2$$
$$x = \underline{426.9\ m}$$

(c) $v_i = 0$

$a = 3.6 m/s^2$

$t = 15.4 s$

$d = 426.9 m$

$v_f = ?$

$$v_f = v_i + at$$
$$= (3.6)(15.4)$$
$$= \underline{55.44\ m/s}$$

55.44 m/s ≅ 200 km/h.
so this is not really realistic

(11) (a) One car ends up being 25 m ahead of the other
since car A has a faster acceleration, it will have traveled farther than car B.

Car A	Car B
$v_i = 0$	$v_i = 0$
$a = 2.5\ m/s^2$	$a = 2.0\ m/s^2$
$d = x + 25$	$d = x$
$t = ?$	$t = ?$

time will be the same

$$d = v_i t + \tfrac{1}{2}at^2$$
$$x + 25 = \tfrac{1}{2}(2.5)t^2$$
$$\frac{x+25}{1.25} = t^2$$

$$d = v_i t + \tfrac{1}{2}at^2$$
$$x = \tfrac{1}{2}(2)t^2$$
$$t^2 = x$$

since time is the same

$$\left(\frac{x+25}{1.25}\right) = x$$

solve for x

$$1.25\left(\frac{x+25}{1.25}\right) = 1.25x$$

$$25 = 0.25x$$
$$x = 100$$

now solve ~~for~~ t

from above $t^2 = x$

$$t^2 = 100$$
$$t = 10\ s$$

(b) We need to calculate time for Car B and then use that to find the speed of Car A

Car B

$v_i = 0$
$a = 2.0\ m/s^2$
$v_f = 14\ m/s$
$t = ?$

$v_f = v_i + at$
$14 = 2t$
$t = 7.0\ s$

Car A

$v_i = 0$
$a = 2.5\ m/s^2$
$t = 7.0\ s$
$v_f = ?$

$v_f = v_i + at$
$= (2.5)7$
$v_f = \underline{17.5\ m/s}$

(12) (a) to catch the speeder, both cars must have traveled the same distance in the same amount of time

Speeder

$v = 100\ km/h = 27.78\ m/s$
$d = x$
$t = ?$

police

$v_i = 0$
$a = 3.6\ m/s^2$
$d = x$
$t = ?$

both displacement and time are the same.

$$v = \frac{d}{t}$$

$$27.78 = \frac{x}{t}$$

$$27.78 = \frac{1.8t^2}{t}$$

$$\underline{t = 15.4\ s}$$

$$d = v_i + \frac{1}{2}at^2$$

$$x = \frac{1}{2}(3.6)t^2$$

$$x = 1.8t^2$$